

Find the co-ordinates of the vertices and foci, and the equations of the asymptotes of the hyperbola
 $x^2 - 4y^2 + 6x - 32y - 19 = 0$. State clearly which co-ordinates are for which points.

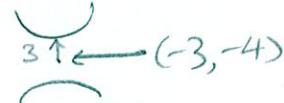
SCORE: ____ / 8 PTS

$$x^2 + 6x - 4y^2 - 32y = 19$$

$$x^2 + 6x + 9 - 4(y^2 + 8y + 16) = 19 + 9 - 4 \cdot 16$$

$$(x+3)^2 - 4(y+4)^2 = -36 \quad (2)$$

$$\frac{(y+4)^2}{9} - \frac{(x+3)^2}{36} = 1 \quad (1)$$



$$c^2 = 9 + 36 = 45 \\ c = 3\sqrt{5}$$

CENTER $(-3, -4)$

VERTICES $(-3, -4 \pm 3) = (-3, -1), (-3, -7)$ (1½)

FOCI $(-3, -4 \pm 3\sqrt{5})$ (1½)

ASYMPTOTES SLOPE = $\pm \frac{\sqrt{9}}{\sqrt{36}} = \pm \frac{3}{6} = \pm \frac{1}{2}$

$$y + 4 = \pm \frac{1}{2}(x + 3) \quad (1)$$

Classify the graph of each equation as a circle, a parabola, an ellipse or a hyperbola.

SCORE: ____ / 2 PTS

[a] $3x^2 + 3x - 2y^2 + 4y + 1 = 0$ HYPERBOLA (1)

[b] $7x^2 + 8x + 12y - 2 = 0$ PARABOLA (1)

A point has polar coordinates $\left(14, \frac{7\pi}{9}\right)$.

SCORE: ____ / 2 PTS

[a] Find another set of polar coordinates for the point, using a positive value of r .

$$(14, \frac{7\pi}{9} \pm 2\pi) = (14, \frac{25\pi}{9}) \text{ OR } (14, \frac{-11\pi}{9})$$

(1) IF YOU GOT EITHER ANSWER

[b] Find another set of polar coordinates for the point, using a negative value of r .

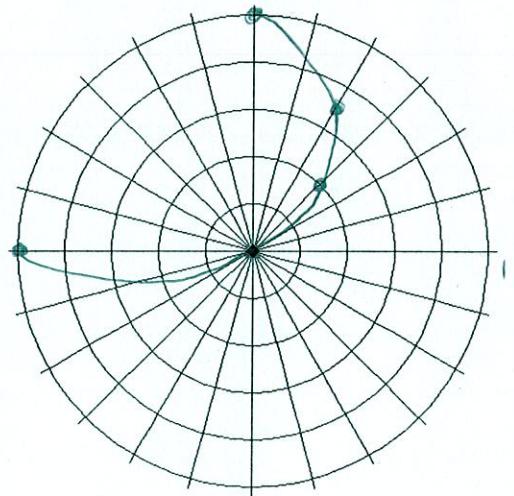
$$(-14, \frac{7\pi}{9} \pm \pi) = (-14, \frac{16\pi}{9}) \text{ OR } (-14, \frac{-2\pi}{9})$$

(1) IF YOU GOT EITHER ANSWER

Plot the graph of $r = 2 \sin \theta - 1$ for $0 \leq \theta \leq \frac{\pi}{2}$. You must list the polar coordinates for 5 points in that range. SCORE: ____ / 8 PTS
Use decimal approximations for irrational values of r .

POINTS ON GRAPH

- (-1, 0)
 (0, $\frac{\pi}{6}$)
 (0.4, $\frac{\pi}{4}$)
 (0.7, $\frac{\pi}{3}$)
 (1, $\frac{\pi}{2}$)



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Convert the rectangular coordinates $(3\sqrt{3}, -9)$ to polar coordinates.

SCORE: ____ / 4 PTS

$$r = \sqrt{27+81} = \sqrt{108} = 6\sqrt{3}$$

$$\sin \theta = -\frac{9}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{9\sqrt{3}}{18} = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

$$\left. \begin{array}{l} \text{① IF IN CORRECT ORDER} \\ \theta = \frac{5\pi}{3} \\ (6\sqrt{3}, \frac{5\pi}{3}) \\ \text{①} \\ \text{②} \end{array} \right\}$$

Convert the polar equation $r^2 = 5 + \sin 2\theta$ to rectangular form.

SCORE: ____ / 6 PTS

$$r^2 = 5 + 2 \sin \theta \cos \theta \quad \text{②}$$

$$r^2 = 5 + \frac{2xy}{r^2} \quad \text{①+}$$

$$r^4 = 5r^2 + 2xy \quad \text{①} \leftarrow \text{CAN ALSO BE WRITTEN AS}$$

$$(x^2+y^2)^2 = 5(x^2+y^2) + 2xy$$

$$(x^2+y^2)(x^2+y^2-5) = 2xy \quad \text{①+} \quad \text{IF YOU GOT EITHER ANSWER}$$

$$r^2(r^2-5) = 2xy$$